CHAPTER 8 Study Guide and Review

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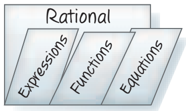
Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Rational Expressions (Lessons 8-1 and 8-2)

- Multiplying and dividing rational expressions is similar to multiplying and dividing fractions.
- To simplify complex fractions, simplify the numerator and the denominator separately, and then simplify the resulting expression.

Direct, Joint, and Inverse Variation (Lesson 8-4)

- Direct Variation: There is a nonzero constant k such that $y = kx$.
- Joint Variation: There is a number k such that $y = kxz$, where $x \neq 0$ and $z \neq 0$.
- Inverse Variation: There is a nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$.

Classes of Functions (Lesson 8-5)

- The following functions can be classified as special functions: constant function, direct variation function, identity function, greatest integer function, absolute value function, quadratic function, square root function, rational function, inverse variation function.

Rational Equations and Inequalities (Lesson 8-6)

- Eliminate fractions in rational equations by multiplying each side of the equation by the LCD.
- Possible solutions of a rational equation must exclude values that result in zero in the denominator.

Key Vocabulary

asymptote (p. 457)
complex fraction (p. 445)
constant of variation (p. 465)
continuity (p. 457)
direct variation (p. 465)
inverse variation (p. 467)
joint variation (p. 466)
point discontinuity (p. 457)
rational equation (p. 479)
rational expression (p. 442)
rational function (p. 457)
rational inequality (p. 483)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. The equation $y = \frac{x^2 - 1}{x + 1}$ has a(n) <u>asymptote</u> at $x = -1$.
2. The equation $y = 3x$ is an example of a(n) <u>direct</u> variation equation.
3. The equation $y = \frac{x^2}{x + 1}$ is a(n) <u>polynomial</u> equation.
4. The graph of $y = \frac{4}{x - 4}$ has a(n) <u>variation</u> at $x = 4$.
5. The equation $b = \frac{2}{a}$ is a(n) <u>inverse</u> variation equation.
6. On the graph of $y = \frac{x - 5}{x + 2}$, there is a break in continuity at $x = \underline{2}$.
7. The expression $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$ is an example of a <u>complex fraction</u>.
8. In the direct variation $y = 6x$, 6 is the <u>degree</u>.

Lesson-by-Lesson Review

8–1 Multiplying and Dividing Rational Expressions (pp. 442–449)

Simplify each expression.

9. $\frac{-4ab}{21c} \cdot \frac{14c^2}{22a^2}$

10. $\frac{a^2 - b^2}{6b} \div \frac{a + b}{36b^2}$

11. $\frac{\frac{x^2 + 7x + 10}{x + 2}}{\frac{x^2 + 2x - 15}{x + 2}}$

12. $\frac{\frac{1}{n^2 - 6n + 9}}{\frac{n + 3}{2n^2 - 18}}$

13. $\frac{y^2 - y - 12}{y + 2} \div \frac{y - 4}{y^2 - 4y - 12}$

14. $\frac{x^2 + 3x - 10}{x^2 + 8x + 15} \cdot \frac{x^2 + 5x + 6}{x^2 + 4x + 4}$

15. GEOMETRY A triangle has an area of $2x^2 + 4x - 16$ square meters. If the base is $x - 2$ meters, find the height.

Example 1 Simplify $\frac{3x}{2y} \cdot \frac{8y^3}{6x^2}$.

$$\frac{3x}{2y} \cdot \frac{8y^3}{6x^2} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{y}} \cdot y \cdot y}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{x}} \cdot x}$$

$$= \frac{2y^2}{x}$$

Example 2 Simplify $\frac{p^2 + 7p}{3p} \div \frac{49 - p^2}{3p - 21}$.

$$\frac{p^2 + 7p}{3p} \div \frac{49 - p^2}{3p - 21} = \frac{p^2 + 7p}{3p} \cdot \frac{3p - 21}{49 - p^2}$$

$$= \frac{\overset{1}{\cancel{p}}\overset{1}{\cancel{(7 + p)}}}{\underset{1}{\cancel{3}}\underset{1}{\cancel{p}}} \cdot \frac{-\overset{1}{\cancel{3}}\overset{1}{\cancel{(7 - p)}}}{\underset{1}{\cancel{(7 + p)}}\underset{1}{\cancel{(7 - p)}}}$$

$$= -1$$

8–2 Adding and Subtracting Rational Expressions (pp. 450–456)

Simplify each expression.

16. $\frac{x + 2}{x - 5} + 6$

17. $\frac{x - 1}{x^2 - 1} + \frac{2}{5x + 5}$

18. $\frac{7}{y} - \frac{2}{3y}$

19. $\frac{7}{y - 2} - \frac{11}{2 - y}$

20. $\frac{3}{4b} - \frac{2}{5b} - \frac{1}{2b}$

21. $\frac{m + 3}{m^2 - 6m + 9} - \frac{8m - 24}{9 - m^2}$

BIOLOGY For Exercises 22 and 23, use the following information.

After a person eats something, the pH or acid level A of their mouth can be determined by the formula $A = -\frac{20.4t}{t^2 + 36} + 6.5$, where t is the number of minutes that have elapsed since the food was eaten.

22. Simplify the equation.

23. What would the acid level be after 30 minutes?

Example 3 Simplify $\frac{14}{x + y} - \frac{9x}{x^2 - y^2}$.

$$\frac{14}{x + y} - \frac{9x}{x^2 - y^2} = \frac{14}{x + y} - \frac{9x}{(x + y)(x - y)}$$

$$= \frac{14(x - y)}{(x + y)(x - y)} - \frac{9x}{(x + y)(x - y)}$$

$$= \frac{14(x - y) - 9x}{(x + y)(x - y)}$$ Subtract the numerators.

$$= \frac{14x - 14y - 9x}{(x + y)(x - y)}$$ Distributive Property

$$= \frac{5x - 14y}{(x + y)(x - y)}$$ Simplify.

Mixed Problem Solving
For mixed problem-solving practice, see page 801.

8–3 Graphing Rational Functions (pp. 457–463)

Graph each rational function.

24. $f(x) = \frac{4}{x - 2}$

25. $f(x) = \frac{x}{x + 3}$

26. $f(x) = \frac{2}{x}$

27. $f(x) = \frac{x^2 + 2x + 1}{x + 1}$

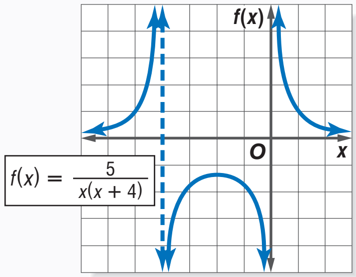
28. $f(x) = \frac{x - 4}{x + 3}$

29. $f(x) = \frac{5}{(x + 1)(x - 3)}$

30. SANDWICHES A group makes 45 sandwiches to take on a picnic. The number of sandwiches a person can eat depends on how many people go on the trip. Write and graph a function to illustrate this situation.

Example 4 **Graph $f(x) = \frac{5}{x(x + 4)}$.**

The function is undefined for $x = 0$ and $x = -4$. Since $\frac{5}{x(x + 4)}$ is in simplest form, $x = 0$ and $x = -4$ are vertical asymptotes. Draw the two asymptotes and sketch the graph.



8–4 Direct, Joint, and Inverse Variation (pp. 465–471)

31. If y varies directly as x and $y = 21$ when $x = 7$, find x when $y = -5$.

32. If y varies inversely as x and $y = 9$ when $x = 2.5$, find y when $x = -0.6$.

33. If y varies inversely as x and $y = -4$ when $x = 8$, find y when $x = -121$.

34. If y varies jointly as x and z and $x = 2$ and $z = 4$ when $y = 16$, find y when $x = 5$ and $z = 8$.

35. If y varies jointly as x and z and $y = 14$ when $x = 10$ and $z = 7$, find y when $x = 11$ and $z = 8$.

36. EMPLOYMENT Chris's pay varies directly with how many lawns he mows. If his pay is \$65 for 5 yards, find his pay after he has mowed 13 yards.

Example 5 **If y varies inversely as x and $x = 14$ when $y = -6$, find x when $y = -11$.**

$\frac{x_1}{y_2} = \frac{x_2}{y_1}$ — Inverse variation

$\frac{14}{-11} = \frac{x_2}{-6}$ — $x_1 = 14, y_1 = -6, y_2 = -11$

$14(-6) = -11(x_2)$ — Cross multiply.

$-84 = -11x_2$ — Simplify.

$7\frac{7}{11} = x_2$ — Divide each side by -11.

When $y = -11$, the value of x is $7\frac{7}{11}$.

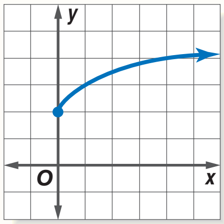
CHAPTER 8

Study Guide and Review

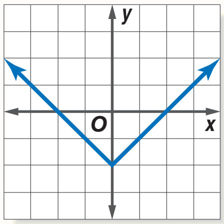
8–5 Classes of Functions (pp. 473–478)

Identify the type of function represented by each graph.

37.

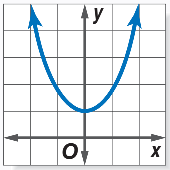


38.



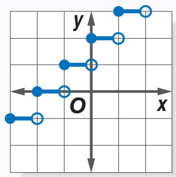
Example 6 Identify the type of function represented by each graph.

a.



The graph has a parabolic shape; therefore, it is a quadratic function.

b.



The graph has a stair-step pattern; therefore, it is a greatest integer function.

8–6 Solving Rational Equations and Inequalities (pp. 479–486)

Solve each equation or inequality. Check your solutions.

39. $\frac{3}{y} + \frac{7}{y} = 9$

40. $\frac{3x + 2}{4} = \frac{9}{4} - \frac{3 - 2x}{6}$

41. $\frac{1}{r^2 - 1} = \frac{2}{r^2 + r - 2}$

42. $\frac{x}{x^2 - 1} + \frac{2}{x + 1} = 1 + \frac{1}{2x - 2}$

43. $\frac{1}{3b} - \frac{3}{4b} > \frac{1}{6}$

44. PUZZLES Danielle can put a puzzle together in three hours. Aidan can put the same puzzle together in five hours. How long will it take them if they work together?

Example 7 Solve $\frac{1}{x - 1} + \frac{2}{x} = 0$.

The LCD is $x(x - 1)$.

$$\frac{1}{x - 1} + \frac{2}{x} = 0$$

$$x(x - 1)\left(\frac{1}{x - 1} + \frac{2}{x}\right) = x(x - 1)(0)$$

$$x(x - 1)\left(\frac{1}{x - 1}\right) + x(x - 1)\left(\frac{2}{x}\right) = x(x - 1)(0)$$

$$1(x) + 2(x - 1) = 0$$

$$x + 2x - 2 = 0$$

$$3x - 2 = 0$$

$$3x = 2$$

$$x = \frac{2}{3}$$